647510 **0301207C1010** **Seat No: ___________**

B.Sc. Semester - 2 (CBCS) Examination
March/April- 2018
MATHEMATICS
(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) Answer the following questions briefly. (04)

(1) Find the centre and radius of the sphere $x^2+y^2+z^2 = 9$.

(2) Find the equation of sphere of which extremities of diameter are (1,1,0) and (0,1,1)

(3) Write the equation of cylinder whose axis is parallel to Y-axis and radius r.

(4) Define right circular cylinder.

Que-1 (B) Answer any One. (02)

(1) Find the two tangent planes to the sphere $x^2+y^2+z^2-6x-4y-2z+5=0$ which are parallel to the plane $8x+y+4z=0$.

(2) Find the radius of the right circular cylinder whose guiding curve is a circle $x^2+y^2+z^2=4$; $x+y+z=3$.

Que-1 (C) Answer any One. (03)

(1) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

(2) Find equation of right circular cylinder passing through (3,4,5) with radius 2 and axis is parallel to {(3,4,5+K)|KER}.

Que-1 (D) Answer any One. (05)

(1) Obtain the equation of tangent plane at (α, β, r) of the sphere $x^2+y^2+z^2+2yx+2vy+2wz+d=0$.

(2) Derive equation of a cylinder of which generator remain parallel to the line $x/l = y/m = z/n$ and passing through a guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0$; $z=0$.

Que-2 (A) Answer the following questions briefy. (04)

(1) Define circular nbhd.

(2) Obtain iterated $\lim\limits_{x\to 0}\left\{\lim\limits_{y\to 0} \frac{\sin(x+y)}{(x+y)}\right\}$.

(3) If $x=r\cos\theta$ then find $\frac{\partial y}{\partial r}$.

(4) If $u=\log(x^2+y^2)$ then find $\frac{\partial u}{\partial x}$.

Que-2 (B) Answer any One. (02)

(1) Using definition of limit prove that $\lim\limits_{(x,y)\to (1,z)} xy=2$.

(2) If $u=\log(\tan x+\tan y)$ then prove that $\sin 2x.\frac{\partial u}{\partial x} + \sin 2y.\frac{\partial u}{\partial y} = 2$.

Que-2 (C) Answer any One. (03)

(1) If Z is a homogeneous function of x & y of degree n and z=f(u) than prove $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = n.\frac{f(u)}{f'(u)}$.

(2) If u = $log\left(\frac{x^2+y^2}{x+y}\right)$ then prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = 1$.

Que-2 (D) Answer any one. (05)

(1) State and prove Euler's theorem for homogeneous function of two variables.

(2) If u=Ø(x,y) where x=e^a cost and y=e^a sint, then show that
$\frac{\partial^2 u}{\partial x^2} + y.\frac{\partial^2 u}{\partial y^2} = e^{-2a}\left[\frac{\partial^2 u}{\partial a^2} + y.\frac{\partial^2 u}{\partial t^2}\right]$.

Que-3 (A) Answer the following questions briefly. (04)

(1) Define : extreme point.

(2) Define : Local maxima and minima.

(3) If x=r $\cos\theta$ and y=r $\sin\theta$. then find $J\left(\frac{x,y}{r,\theta}\right)$

(4) $\frac{\partial(u,v)}{\partial(r,s)} \cdot \frac{\partial(r,s)}{\partial(x,y)} =$ ……………………….

Que-3 (B) Answer any One. (02)

(1) If f(x,y) = x^2y - 3y then fine the approximate value of f(5:12, 6.85).

(2) If u=x^2-3y, v=x+y+z, w=x-2y+3z then show that $\frac{\partial(u,v,w)}{\partial(x.y,z)}$ = 10x + 4.

Que-3 (C) Answer any One. (03)

(1) Find the maxima and minima of the function f(x,y) = x^3+y^3-3xy.

(2) If there is 0.05% error obtain in measurement of length of rectangle then what should be an error in the measurement of its area.

Que-3 (D) Answer any One. (05)

(1) Expand e^x cosy in powers of x and y upto three degree using Taylor's theorem.

(2) If u $=\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ then the extreme value of u when lx+my+nz=0 and $\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ =1 can be obtained by an equation $\frac{l^2a^4}{1-a^2u} + \frac{m^2b^4}{1-b^2u} + \frac{n^2c^4}{1-c^2u}$ = 0.

Que-4 (A) Answer the following questions briefly. (04)

(1) Find trace of matrix A=$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(2) For square matrix A, if A^5=A then write the period of A.

(3) If for square matrix A, A^7=zero matrix then find index of matrix A.

(4) Define : Orthogonal matrix.

Que-4 (B) Answer any One. (02)

(1) Prove : $(AB)^T$=B^T.A^T.

(2) Check A=$\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is Milpotent or not ? also find its Index.

Que-4 (C) Answer any One. (03)

(1) Find the rank of the matrix
A=$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

(2) Prove that A=1/3 $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is an orthogonal matrix.

Que-4 (D) Answer any One. (05)

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

(2) Find the rank of A=$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ using echelon matrix.

Que-5 (A) Answer the following questions briefly (04)

(1) Define : Eigen vectors.

(2) Find the characteristic equation of $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(3) Find the eigen values of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

(4) Define : Augmented matrix.

Que-5 (B) Answer any One. (02)

(1) Prove that 0 is a characteristic root of a Matrix iff the Matrix is singular.

(2) Matrix A-λI is singular iff λ is an Eigen value of Matrix A.

Que-5 (C) Answer any One. (03)

(1) Solve : x+y+z=9

2x+5y+7z=52

2x+y-z=0

(2) Prove that there is unique Eigen value with respect to any Eigen vector of given Matrix.

Que-5 (D) Answer any One. (05)

(1) State and prove cauley - Hamilton theorem.

(2) Find eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

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